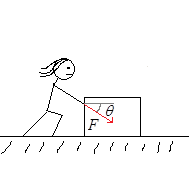
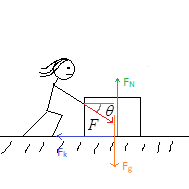
**Homework 9 Solutions**

**Problem 1.** Suppose Cinderella pushes a 45 kg box of slippers along a floor as shown below, with a force F = 200N directed at an angle θ = 29◦ below the horizontal. If she pushes the box through a distance of 24m, what will be its speed at that point? Let there be a kinetic friction coefficient μk = 0.17.



So we have to do the work-energy theorem. First we should label all the forces acting on the box:



And calculate the work done by the non-conservative ones. Every force is non-conservative except for the two potential energy forces: gravity and spring. So we must calculate WF, WFk and WFN.



So to get FN we use N2L in the y-direction.



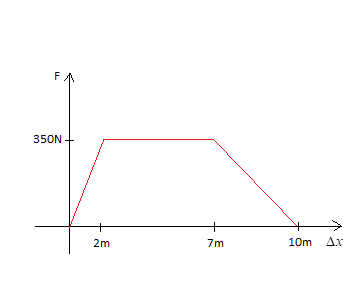
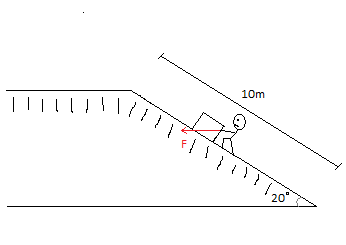
So then,



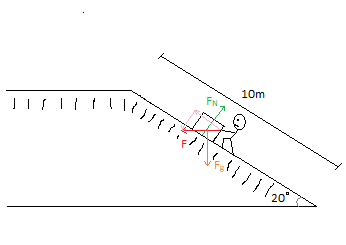
So then we fill this into the WE equation:



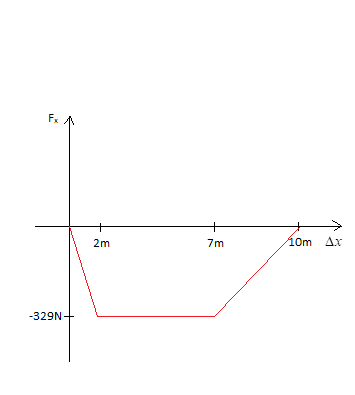
**Problem 2.** Now she shoves the box onto a descending frictionless ramp with an initial speed 7m/s. The prince slows it down by pushing with a horizontal force, whose magnitude changes with position along the ramp, according to the graph shown below. What is the box’s speed at the bottom of the ramp?



Let’s draw the forces acting on the box again (I’ve split F’s force into its components parallel and perpendicular to the plane).



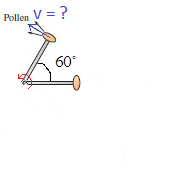
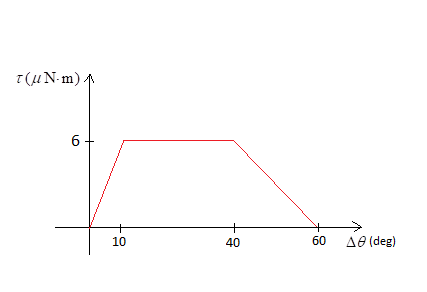
F and FN are the non-conservative ones, and so we must calculate their work. Of course FN, being perpendicular to the displacement, does no work, as usual. As for F, if we align our x axis with the plane, then it is just WF = Area under Fx vs. Δx curve. Fx = -Fcos(20°) (- sign is because it’s going in opposite direction of displacement Δx), and would have a maximum value of -350cos(20°) = -329N. And so the graph of Fx vs. Δx would look like:



And so the work would be: W = area = bave∙h = (10+5)/2∙(-329) = -2470J. Now let’s fill it into the WE equation:



**Problem 3.** A bunchberry flower catapults its pollen to aid in reproduction. Suppose the flower exerts the torque graphed below on a stamen with anther sac/pollen attached (note μN∙m = 10-6N∙m). After the stamen has rotated through 60°, it releases the anther sac. You can model the anther sac as a point mass manther = 10μg = 10×10-9kg, and the stamen as a rod of mass mstamen = 15μg = 15×10-9kg, and length ℓ = 1.3mm. Don’t forget to include the effects of gravity, and calculate with what speed the anther sack is released.

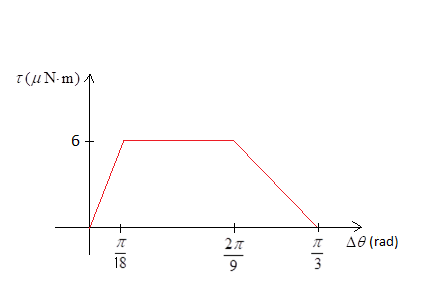
 

So we have:



We’re going to have to get the work done by the torque rotating the stamen+anther sac. This is:

Wτ = area under τ vs. Δθ curve. But θ must be in radians. So converting the angles to radians: θrad = (π/180)∙θ°, we have:



The work is then:



Moving on the potential energy terms, note that when calculating gravitational potential energy, we need to measure height to the middle of the object. So



The moment of the inertia of the stamen must be obtained from the parallel axis theorem. It’s moment of inertia about its center is Icm = (1/12)mstamenℓ2, and so its moment of inertia about its endpoint would be I = Icm + mstamenh2 = (1/12)mstamenℓ2 + mstamen(ℓ/2)2 = (1/3)mstamenℓ2. We don’t know the angular velocity of the stamen either. But it can be related to our primary unknown, v, via the relationship v = ωr → ω = v/r = v/ℓ. So altogether then, we have:



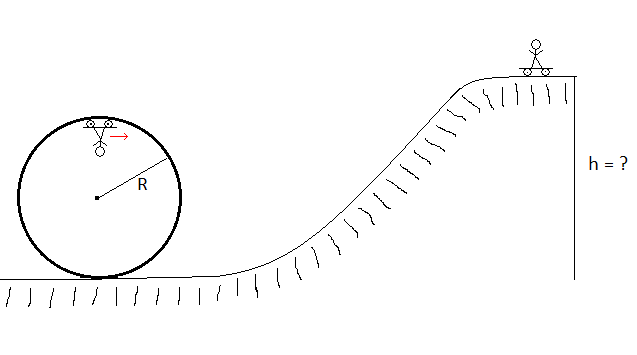
And then for the anther sac we have:



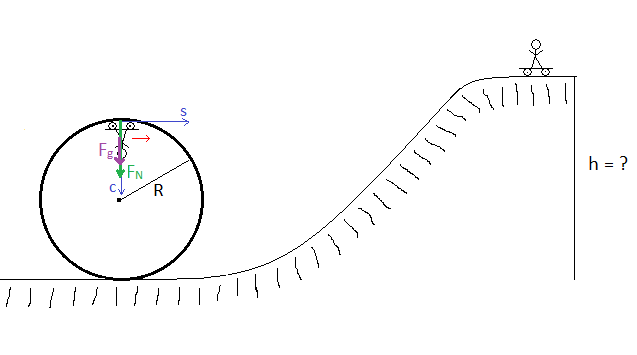
So putting it all together,



**Problem 4.** You want to ride your skateboard through a loop-the-loop of radius R = 5m, so you build a ramp h meters high to start from. Note that if you made h = 2R (i.e. the same height as the loop-the-loop), then you would get to the top of the loop-the-loop, but since your speed at the top would then be zero, you’d immediately fall of the track. So h needs to be higher, high enough so that you will continue to go around the loop when you get to the top. So what is h? You can ignore friction.



First we have to calculate the minimum speed you need at the top of the loop. This is the speed for which you are just barely staying on the track, and this is the one for which FN is just barely above 0. I’ve drawn the forces at the top, as well as the centripetal and tangential axes:



We need:

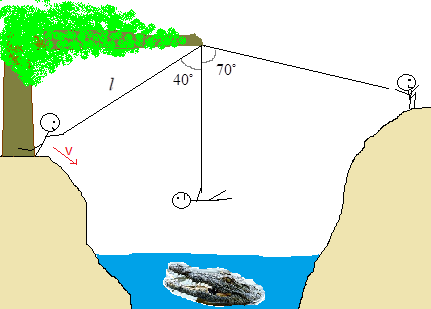


And now we use energy conservation between the top of the ramp and the top of the loop,



So h = 2.5(5) = 12.5m.

**Problem 5.** You’re got to swing across a crocodile infested river with an ℓ = 10m long vine. Say that you have a mass of 70kg. What minimum speed do you need to leap off the cliff with, so that you just barely make it to the top on the other side? If you can only grip onto the vine with a maximum force of twice your body weight, would you slip off the vine or not?



We’ll apply the work-energy equation:



During your swing, the only non-conservative force acting on you would be tension. But it, like the normal force, always acts perpendicular to your motion when you’re executing a circle. And so the work it does is 0, just like the normal force. Next, for these problems it’s most convenient to measure position from the center of the circle. So let’s take y = 0 to be where the rope is tired to the tree – that will make all of our y’s negative. So we have:



Then to get ascertain whether you’d fall off or not, we should look at where the tension in the vine would be largest. This would be at the bottom of the swing. And we’d use N2L in the centripetal direction:



But we need the speed at the bottom of the swing so we use energy conservation again,

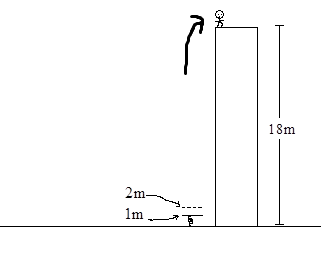


And now plugging this back into the energy conservation equation to FT:



So the tension in the vine (and in your hands/arms by extension) would have to be 1580N, which is greater than twice your weight 2mg = 2(70)(9.8) = 1370N. So you’d fall off into the river and die. Unless you could wrap the vine around your hand or something, so you wouldn’t have to *hold* onto the vine per se´.

**Problem 6.** Suppose you construct a spring-loaded contraption to propel you to the top of a building 18m high. Your mass is m = 65kg, and you plan on compressing the spring 1m from its rest length of 2m so that you will start 1m off the ground. What must the spring constant of the spring be? What would be your maximum speed? And what would be your maximum acceleration?



We have:



Your maximum speed would occur at the ‘equilibrium point’ of the spring – the point where the gravitational force on you, matches the spring force on you. This is where Fg = Fspring → mg = kx → (65)(9.8) = (21700)x → x = 0.03m = 3cm. So the spring would be compressed only 3cm at the equilibrium point. And then your velocity here would be:



Maximum acceleration would be at the bottom, and this would be given by:



This is 33 times the acceleration due to gravity. So you’d probably pass out. Bettter make a longer spring so you don’t have to accelerate as quickly.

**Problem 7.** Suppose you have a mass of 65kg, your bike frame has a mass of 10kg, and the tires on your bike each have a mass of 5kg, and a diameter of 80cm. If you coast down a hill 15m high, how fast will you be going at the bottom? You can treat the tires as hoops.



Work energy equation…this time the non-conservative forces are the normal force and static friction forces the ground exerts on your tires. But these also don’t do work. The normal force doesn’t for the reason that it is perpendicular to your motion. The static friction force doesn’t for a more subtle reason. The bottom of the wheel, where the friction force acts is actually instantaneously at rest (vbottom of wheel = 0) when it is rolling because the center of the wheel is traveling with velocity vcenter = -v (down the hill), but the bottom of the wheel is rotating about the center with speed vabout the center = ωr = (+v/r)r = +v. And so the net velocity of the bottom of the wheel is vbottom of wheel = vcenter + vabout the center = -v + v = 0. Which means it is not moving at *the* instant friction acts on it. And if it isn’t moving, then no work is being done on it. So the upshot is we have energy conservation:



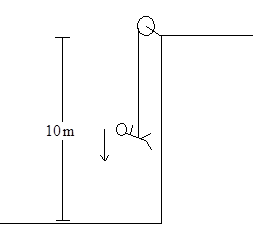
Note Iwheels =2Iwheel = 2∙MwheelR2 = 2∙(5)(0.40)2 = 1.6kg∙m2. So then



Then we can say that ωf = vf/R = vf/0.40. So,



**Problem 8.** Suppose you drop off a rock wall, your rope attached to a pulley, shaped like a disk. As you fall, the rope will spin the disk (like a fish pulling on the reel of a fishing pole) which will in turn slow your rate of decent. Say the mass of the disk is M = 200kg, and your mass is m = 70kg. If you drop from rest at the top of the wall, what will be your speed when you hit the floor?



Using WE equation….and note the moment of inertia of the disk is I = (1/2)MR2 = (1/2)(200)R2 = 100R2 – no radius given, so just treat it as a variable…



But note ωdisk = vyou/R, so:



In contrast, if you just stepped off the wall, you’d hit the ground at 14m/s.

**Problem 9.** Your 3000kg car is driving up a 15° incline at a constsant speed of 25m/s. What power must the engine deliver to the wheels to accomplish this?

Power is P = dW/dt. And it is often easiest to use the equivalence of work and energy, to calculate power. So by the work energy equation, we can say P = dE/dt. The energy of your car is: (1/2)mv2 + mgy. But your speed isn’t changing so, P = d(mgy)/dt = mg∙dy/dt = mgvy. And vy = 25sin(15°) = 6.5m/s So P = (3000kg)(9.8m/s2)(6.5m/s) = 191 000W = 191kW = 260h.p.

**Problem 10.** You’re on a stair climber making one step of 30cm every second. Your mass is 60kg. What power must you deliver to your muscles to accomplish the exercise? Your metabolic efficiency is about 20%, so what is your metabolic power output? How many kcal would you ‘burn’ in 30 minutes of such exercise?

Again, P = dE/dt = d(mgy)/dt = mg∙dy/dt = mg∙(30cm/second) = (60kg)(9.8m/s2)(0.30m/s) = 176W.

Your metabolic power output woud be Pmetabolic = P/efficiency = 176W/0.20 = 880W.

Your total metabolic energy output in 30 minutes would be ΔE = PΔt → (880W)(30∙60s) = 1.58×106J. And 1kcal = 4180J, so this corresponds to: ΔE = 1.58×106/4180 = 379kcal.

**Problem 11.** Water flows over Niagara falls at a rate of 1800 m3/s down a distance of approximately 45m. The water obviously picks up kinetic energy as it falls. But when it hits the rocks below, all the kinetic energy gets dispersed and wasted. Electrical power plants are often built near waterfalls to instead utilize this kinetic energy to turn turbines which will generate electrical power via electromagnetic induction (PHY 123). What would be the power output of a plant utilizing all the water in Niagra falls? Compare this to the power requirements of a medium size city (P ~ 1GW). Note the density of water is ρ = 1000kg/m3. And you can assume the water is nearly stationary at the top of the falls, for simplicity.

So P = ΔE/Δt. And every second 1800m3 (mass = ρV = 1000k/m3∙1800m3 = 1.8×106kg) of water flows over the falls, and by the time it gets to the bottom of the falls, will have acquired a kinetic energy KE equal to its change in gravitational potential energy PEg = mgy = 1.8×106kg∙9.8m/s2∙45m = 794×106J. So normally every second this energy gets dispersed, but if the plant utilizes it instead, then it would generate a power output of P = ΔE/Δt = 794×106J/1s = 794×106W = 794MW = 0.794GW. This alone would almost be enough to power the city.